

A note on R -currents and trace anomalies in the (2,0) tensor multiplet in $d = 6$ and AdS/CFT correspondence

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Abstract

We discuss the two- and three-point functions of the $SO(5)$ R -current of the (2,0) tensor multiplet in $d = 6$, using AdS/CFT correspondence as well as a free field realization. The results obtained via AdS/CFT correspondence coincide with the ones from a free field calculation up to an overall $4N^3$ factor. This is the same factor found recently in studies of the two- and three-point functions of the energy momentum tensor in the (2,0) theory. We connect our results to the trace anomaly in $d = 6$ in the presence of external vector fields and briefly discuss their implications.

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Correlation functions of conserved currents carry important information for a quantum field theory. In the case of AdS/CFT correspondence [1], their study has unveiled important properties of the boundary CFT. Particular examples are studies of the two- and three-point functions of the energy momentum tensor [2–5] and also of the two- and three-point functions of conserved vector currents [6, 7]. These have provided important tests, both for the AdS/CFT correspondence and also for the non-renormalization properties of $\mathcal{N} = 4$ SYM in $d = 4$. Recently, there has been a growing interest for the study of AdS/CFT correspondence in diverse dimensions, such as $\text{AdS}_{4/7}/\text{CFT}_{3/6}$, which are connected with M2/M5 branes respectively. The two- and three-point functions of the energy momentum tensor have been recently discussed in both the above cases [4, 5]. Such studies are of particular interest as the boundary CFTs are not well understood. Our aim here is to extend the results of [4, 5] for the (2,0) multiplet in $d = 6$ by discussing the two- and three-point functions of the $SO(5)$ R -current of the theory, both via $\text{AdS}_7/\text{CFT}_6$ correspondence and also using a free field realization.

It is well known [8] that the general structure of two- and three-point functions in a CFT is fixed by conformal invariance up to a number of constants. Let $J_\mu(x) = J_\mu^A(x)T^A$, $\mu = 1, \dots, d$ be a general conserved current. In the case when $J_\mu(x)$ corresponds to the R -current of the boundary CFT_d obtained via $\text{AdS}_{d+1}/\text{CFT}_d$ correspondence, T^A are the Lie algebra generators of the isometry group of the sphere resulting from the compactification of the corresponding M/IIB theory on $\text{AdS}_{d+1} \times S^{(10/9-d)}$. Then, from conformal invariance we can write in the (flat) boundary theory [8]

$$\langle J_\mu^A(x) J_\nu^B(y) \rangle = \delta^{AB} \frac{\mathcal{C}_V^{(d)}}{[(x-y)^2]^{(d-1)}} I_{\mu\nu}(x-y), \quad I_{\mu\nu}(x) = \delta_{\mu\nu} - 2 \frac{x_\mu x_\nu}{x^2}, \quad (1)$$

$$\begin{aligned} \langle J_\mu^A(x) J_\nu^B(y) J_\lambda^C(z) \rangle &= \frac{f^{ABC}}{(x-y)^{d-2} (x-z)^{d-2} (y-z)^d} \\ &\times I_{\nu\sigma}(x-y) I_{\lambda\rho}(x-z) I_{\sigma\beta}(X) t_{\mu\beta\rho}(X), \end{aligned} \quad (2)$$

$$X_\mu = \frac{(x-y)_\mu}{(x-y)^2} - \frac{(x-z)_\mu}{(x-z)^2},$$

$$t_{\mu\nu\lambda}(X) = \mathcal{A}^{(d)} \frac{X_\mu X_\nu X_\lambda}{X^2} + \mathcal{B}^{(d)} (X_\mu \delta_{\nu\lambda} + X_\nu \delta_{\mu\lambda} - X_\lambda \delta_{\mu\nu}),$$

where f^{ABC} are the Lie algebra structure constants in the representation carried by the currents. The constants $\mathcal{C}_V^{(d)}$, $\mathcal{A}^{(d)}$ and $\mathcal{B}^{(d)}$ are important parameters of the boundary CFT_d . We are interested both in the strong coupling as well as in the free field theory values of these parameters. The strong coupling values follow from $\text{AdS}_{d+1}/\text{CFT}_d$ correspondence³ and have been obtained in

³We consider the Euclidean version of AdS_{d+1} space where $d\hat{x}^\mu d\hat{x}_\mu = \frac{1}{x_0^2} (dx_0 dx_0 + dx^i dx^i)$, with $i = 1, \dots, d$, and $\hat{x}_\mu = (x_0, x_i)$. The boundary of this space is isomorphic to \mathbf{S}^d since it consists of \mathbf{R}^d at $x_0 = 0$ and a single point at $x_0 = \infty$.

[6, 7]. Namely, from the $d+1$ dimensional Lagrangian $\mathcal{L} = -\frac{1}{4g_{SG_{d+1}}^2} \int d^{d+1}\hat{x} \sqrt{g} F_{ij}^A(\hat{x}) F^{A,ij}(\hat{x})$, $i, j = 0, \dots, d$ (without the anomalous Chern-Simons term), one obtains

$$\mathcal{C}_V^{(d)} = \frac{\Gamma(d)(d-2)}{2\pi^{\frac{1}{2}d}\Gamma(\frac{1}{2}d)} \frac{1}{g_{SG_{d+1}}^2}, \quad (3)$$

$$\mathcal{B}^{(d)} = \frac{(2d-3)\Gamma(\frac{1}{2}d)}{\pi^{\frac{1}{2}d}4(d-1)} \frac{\mathcal{C}_V^{(d)}}{g_{SG_{d+1}}^2}, \quad (4)$$

$$\mathcal{A}^{(d)} = \frac{d\Gamma(\frac{1}{2}d)}{\pi^{\frac{1}{2}d}4(d-1)} \frac{\mathcal{C}_V^{(d)}}{g_{SG_{d+1}}^2}. \quad (5)$$

These relations are in agreement with the general conformal Ward identity [8]

$$\mathcal{C}_V^{(d)} = \left(\frac{1}{d} \mathcal{A}^{(d)} + \mathcal{B}^{(d)} \right) S_d, \quad S_d = \frac{2\pi^{\frac{1}{2}d}}{\Gamma(\frac{1}{2}d)}. \quad (6)$$

The crucial strong coupling information provided by (3), (4) and (5) is, apart from the value of $\mathcal{C}_V^{(d)}$, the *relative* scale of $\mathcal{A}_V^{(d)}$ and $\mathcal{B}_V^{(d)}$ which is given by

$$\mathcal{A}^{(d)} = \frac{d}{2d-3} \mathcal{B}^{(d)}. \quad (7)$$

A similar discussion has been presented in [4] for the parameters involved in the two- and three-point functions of the energy momentum tensor. Setting $d = 4, 6$ in (3)-(5) we obtain

$$\mathcal{C}_V^{(4)} = \frac{6}{\pi^2 g_{SG_5}^2}, \quad \mathcal{A}^{(4)} = \frac{2}{\pi^4 g_{SG_5}^2}, \quad \mathcal{B}^{(4)} = \frac{5}{2\pi^4 g_{SG_5}^2}, \quad (8)$$

$$\mathcal{C}_V^{(6)} = \frac{120}{\pi^3 g_{SG_7}^2}, \quad \mathcal{A}^{(6)} = \frac{72}{\pi^6 g_{SG_7}^2}, \quad \mathcal{B}^{(6)} = \frac{108}{\pi^6 g_{SG_7}^2}. \quad (9)$$

The values given in (8) correspond to the large- N , large- $g_{YM}^2 N$ limit (with g_{YM} the gauge coupling), of the $\mathcal{N} = 4$, $d = 4$ $SU(N)$ SYM, while the ones given in (9), presumably, to the strong coupling limit of the (2,0), $d = 6$ selfdual tensor multiplet.

We can calculate the values of the parameters above using a free field realization of the corresponding theories. To obtain the free field (one loop), contributions to these parameters from general scalar and fermion currents with a global symmetry group we follow [8] and define the composite vector current

$$J_\mu^A = \varphi T_\varphi^A \partial_\mu \varphi + \bar{\psi} T_\psi^A \gamma_\mu \psi, \quad (10)$$

$$[T_\varphi^A, T_\varphi^B] = f^{ABC} T_\varphi^C, \quad [T_\psi^A, T_\psi^B] = f^{ABC} T_\psi^C, \quad (11)$$

$$\text{Tr}(T_\varphi^A T_\varphi^B) = -C_\varphi \delta^{AB}, \quad \text{Tr}(T_\psi^A T_\psi^B) = -C_\psi \delta^{AB}. \quad (12)$$

Then, from [8] we obtain in general dimensions

$$\mathcal{C}_{V,free}^{(d)} = \left(\frac{C_\varphi}{d-2} + C_\psi (\text{Tr} \mathbf{I}_\psi) \right) \frac{1}{S_d^2}, \quad (13)$$

$$\mathcal{B}_{free}^{(d)} = \left(\frac{C_\varphi}{2} \frac{1}{d-2} + C_\psi (\text{Tr} \mathbf{I}_\psi) \right) \frac{1}{S_d^3}, \quad (14)$$

$$\mathcal{A}_{free}^{(d)} = \frac{C_\varphi}{2} \frac{d}{d-2} \frac{1}{S_d^3}. \quad (15)$$

Here $(\text{Tr} \mathbf{I}_\psi)$ is the dimension of the spacetime spinors forming the current (10). Note that the constants (13)-(15) do not depend directly on the number of modes but rather on the representations of the global symmetry group under which the scalars and fermions in the theory transform. It is easy to see that the above free field values satisfy the conformal Ward identity (6). Let us then try to reproduce the strong coupling AdS results (8) and (9) using the above free field values. As we have mentioned, the important condition to be satisfied is (7), since the Ward identity (6) implies that the overall normalization of $\mathcal{C}_V^{(d)}$, $\mathcal{A}^{(d)}$ and $\mathcal{B}^{(d)}$ is the same. Substituting then (14) and (15) into (7) we obtain

$$C_\varphi = C_\psi (\text{Tr} \mathbf{I}_\psi). \quad (16)$$

This can be interpreted as a selection rule for the free field multiplets.

Next, using (10)-(15) we can easily evaluate the free field values $\mathcal{C}_V^{(d)}$, $\mathcal{A}^{(d)}$ and $\mathcal{B}^{(d)}$ for the cases of interest to us, namely $\mathcal{N} = 4$ SYM in $d = 4$ with R -symmetry group $SU(4)$ and $(2, 0)$ selfdual tensor multiplet in $d = 6$ with R -symmetry group $SO(5)$. In both cases we have contributions from scalars and chiral fermions.⁴ In the case of $\mathcal{N} = 4$ SYM in $d = 4$ the scalar bosons are in the antisymmetric **6** representation of $SU(4)$ and the chiral fermions in the fundamental representation of $SU(4)$. We then obtain

$$C_\varphi^{\mathbf{6}} = 1, \quad C_\psi^{\mathbf{4}} = \frac{1}{2}, \quad (\text{Tr} \mathbf{I}_\psi) = 2, \quad (17)$$

$$\mathcal{C}_{V,free}^{(4)} = \frac{3}{8\pi^4}, \quad \mathcal{B}_{free}^{(4)} = \frac{5}{32\pi^6}, \quad \mathcal{A}_{free}^{(4)} = \frac{1}{8\pi^6}. \quad (18)$$

Note that the selection rule (16) is satisfied.

For comparison with the results from $\text{AdS}_5/\text{CFT}_4$ correspondence we need the value of $1/g_{S_{G_5}}^2$. This can be obtained for both cases of interest here, $d+1 = 5, 7$, considering the equations of motion of gauged supergravity in 5 and 7 dimensions correspondingly in the presence of non trivial scalar fields in the coset space $SL(n, \mathbf{R})/SO(n)$. The $SO(n)$ group corresponds to the R -symmetry group of the boundary CFT_d . Following [9], the maximally supersymmetric solution of the equations of motion (with the scalar fields set to zero), corresponds to an AdS_{d+1} metric $g_{\mu\nu}$ with negative cosmological constant as

$$R_{\mu\nu} = \frac{4}{d-1} P g_{\mu\nu}, \quad P = -g^2 \frac{n(n-2)}{32}, \quad (n, d+1) = (6, 5), (5, 7), \quad (19)$$

⁴Note that vector fields do not contribute to $\mathcal{C}_V^{(d)}$, $\mathcal{A}^{(d)}$ and $\mathcal{B}^{(d)}$ in $d = 4$ [8].

where g^2 corresponds to the supergravity gauge coupling. Comparing (19) with the vacuum solution $R_{\mu\nu} = -d g_{\mu\nu}$ for IIB/M theory compactifications on $\text{AdS}_{5/7} \times \text{S}^{5/4}$ with AdS radius one, we obtain

$$g^2 = 8 \frac{d(d-1)}{n(n-2)}. \quad (20)$$

Taking then into account the relative $1/2$ normalization between the R and F^2 terms in gauged supergravity [9, 10] we get

$$\frac{1}{g_{SG_{d+1}}^2} = \frac{n(n-2)}{4d(d-1)} \frac{1}{2\kappa_{d+1}^2}. \quad (21)$$

The value of the gravitational constant $1/2\kappa_{d+1}^2$ has been obtained for $d+1 = 5, 7$ in [11, 4] as

$$\frac{1}{2\kappa_5^2} = \frac{N^2}{8\pi^2}, \quad \frac{1}{2\kappa_7^2} = \frac{N^3}{3\pi^3}, \quad (22)$$

where N is the number of coincident $D3/M5$ branes. By virtue of (22) we then obtain for $d+1 = 5$, $n = 6$

$$\frac{1}{g_{SG_5}^2} = \frac{N^2}{16\pi^2}. \quad (23)$$

This value coincides with one obtained in [6, 12]. Finally we obtain

$$\frac{\mathcal{C}_V^{(4)}}{\mathcal{C}_{V,free}^{(4)}} = \frac{\mathcal{B}^{(4)}}{\mathcal{B}_{free}^{(4)}} = \frac{\mathcal{A}^{(4)}}{\mathcal{A}_{free}^{(4)}} = N^2. \quad (24)$$

The overall factor N^2 can be recognized as the large- N value of the dimension of the adjoint representation of $SU(N)$ carried by all fields of $\mathcal{N} = 4$ $SU(N)$ SYM theory. The same N^2 overall factor is obtained in studies of the parameters involved in the two- and three-point functions of the energy momentum tensor in $\mathcal{N} = 4$ SYM in $d = 4$ [2, 3]. The result (24) has been also obtained in [6, 7]. It essentially implies that the quantities $\mathcal{C}_V^{(4)}$, $\mathcal{A}^{(4)}$ and $\mathcal{B}^{(4)}$ are not renormalized as one goes from the weak (free fields) to the strong coupling regime of the theory [15].

Next we perform the same calculation for the $(2, 0)$ supermultiplet in $d = 6$. The scalar fields are realized here as antisymmetric traceless spin-tensors in a spinor representation of the R -symmetry group $SO(5)$ [14]. Namely,

$$\phi^{ij} = -\phi^{ji}, \quad \Omega_{ij}\phi^{ij} = 0, \quad i, j = 1, 2, 3, 4, \quad (25)$$

where Ω_{ij} is the symplectic ($\Omega^T = -\Omega$) invariant tensor of $USp(4)$. The spinors of the $(2, 0)$ supermultiplet form a symplectic Majorana-Weyl spinor as

$$\psi_i = \psi^j \Omega_{ji}. \quad (26)$$

With these fields we construct the R -current with the scalars in the **5** and the spinors in the **4** of $SO(5)$ as follows ⁵

$$J_\mu^{ab} = \phi^{ik} T_{ik,jl}^{ab} \partial_\mu \phi^{jl} + \bar{\psi}^i t_{ij}^{ab} \gamma_\mu \psi^j, \quad J_\mu^{ab} = -J_\mu^{ba} \quad a, b = 1, 2, \dots, 5, \quad (27)$$

$$t_{ij}^{ab} = \frac{1}{4} \gamma_{ij}^{ab}, \quad T_{ik,jl}^{ab} = \frac{1}{8} \left(\gamma_{ij}^{ab} \Omega_{kl} - \gamma_{kj}^{ab} \Omega_{il} + \gamma_{kl}^{ab} \Omega_{ij} - \gamma_{il}^{ab} \Omega_{kj} \right), \quad (28)$$

$$C_\psi^4 = \frac{1}{2}, \quad C_\phi^5 = 1. \quad (29)$$

The ratio $C_\phi^5/C_\psi^4 = 2$ is fixed by the respective representations of scalars and fermions.

Using then (13)-(15) and (25)-(29) we can evaluate the parameters for the (2,0), $d = 6$ supermultiplet. For this we need to take $(\text{Tr} \mathbf{I}_\psi) = 2$ in (13)-(15) which amounts to considering half of the fermionic degrees of freedom [4]. Note that with this prescription the selection rule (16) is satisfied for the values of C_ψ^4 and C_ϕ^5 in (29). We then obtain,

$$\mathcal{C}_{V,free}^{(6)} = \frac{5}{4\pi^6}, \quad \mathcal{B}_{free}^{(6)} = \frac{9}{8\pi^9}, \quad \mathcal{A}_{free}^{(6)} = \frac{3}{4\pi^9}. \quad (30)$$

We can compare the above free field values with the ones obtained via $\text{AdS}_7/\text{CFT}_6$ correspondence in (9), if we substitute from (21) the corresponding value for $1/g_{SG_7}^2$

$$\frac{1}{g_{SG_7}^2} = \frac{N^3}{24\pi^3}. \quad (31)$$

By virtue of (31) we obtain from (9) and (30)

$$\frac{\mathcal{C}_V^{(6)}}{\mathcal{C}_{V,free}^{(6)}} = \frac{\mathcal{B}^{(6)}}{\mathcal{B}_{free}^{(6)}} = \frac{\mathcal{A}^{(6)}}{\mathcal{A}_{free}^{(6)}} = 4N^3. \quad (32)$$

The overall factor $4N^3$ is the same with the one found in studies of the two- and three-point functions of the energy momentum tensor [4].

An important property of the parameters which appear in two- and three-point functions of conserved currents (in even dimensions), is that they are intimately connected to the trace anomaly of the theory in the presence of external sources. For the (2,0) multiplet in $d = 6$, such external sources for the R -current are introduced if one adds to the action the term

$$\int d^d x \sqrt{g} g^{\mu\nu} A_\mu^{ab}(x) J_\nu^{ab}(x). \quad (33)$$

Conservation of the R -current means that the external field is defined up to a gauge transformation $A_\mu^{ab}(x) \sim A_\mu^{ab}(x) + \partial_\mu \alpha^{ab}(x)$. The R -current is in the supermultiplet of conserved currents, which also includes the energy momentum tensor [14]. Therefore, we can consider a

⁵We introduce $USp(4)$ 4×4 gamma matrices γ_{ij}^a and $\gamma_{ij}^{ab} = \frac{1}{2} [\gamma^a, \gamma^b]_{ij}$

supersymmetric introduction of the external source $A_\mu^{ab}(x)$, if we view the latter as a component of a general $d = 6$ background superfield which also includes an external gravitational field. Similar considerations were presented in [15] in the case of four-dimensional theories. Then, a simple extension of the results in [8] allows us to connect all three constants $\mathcal{C}_V^{(d)}$, $\mathcal{A}^{(d)}$ and $\mathcal{B}^{(d)}$ to the trace anomaly of the (2,0) theory in $d = 6$ in the presence of external vector fields. The general idea underlying such a connection is that the external trace anomaly is tied to the short distance singularities of the renormalized n -point functions. In the case of interest here, we can follow [16] and write the general renormalization group equation

$$\begin{aligned} \sum_{k=1}^{\infty} \frac{1}{k!} \int d^6 x_1 \sqrt{g} g^{\mu_1 \nu_1} \dots d^6 x_k \sqrt{g} g^{\mu_k \nu_k} A_{\mu_1}^{a_1 b_1}(x_1) \dots A_{\mu_k}^{a_k b_k}(x_k) \mu \frac{\partial}{\partial \mu} \langle J_{\nu_1}^{a_1 b_1}(x_1) \dots J_{\nu_k}^{a_k b_k}(x_k) \rangle_R \\ = \int d^6 x \sqrt{g} g^{\mu\nu}(x) \langle T_{\mu\nu}(x) \rangle. \end{aligned} \quad (34)$$

The subscript R in the first line of (34) denotes the renormalized n -point functions which depend on the arbitrary mass parameter μ . Taking suitable functional derivatives of (34) with respect to $A_\mu^{ab}(x)$ we can, in principle, connect the parameters which appear in the two- and three-point functions of $J_\mu^{ab}(x)$ with the possible terms in the trace anomaly.

The general structure of the conformal anomaly in $d = 6$ in the presence of external vector fields can be obtained following considerations similar to the ones used in the classification of conformal anomalies in the presence external gravitational fields [13]. In the flat space limit and up to total derivatives there exist only *two* independent gauge and scale invariant contributions to the external trace anomaly. Here we choose to parametrize the anomaly as

$$\langle T_\mu^\mu(x) \rangle = \alpha_V F_\mu^{ab, \nu} F_\nu^{cd, \lambda} F_\lambda^{pq, \mu} f^{ab, cd, pq} + \beta_V \partial^\mu F_{\mu\nu}^{ab} \partial^\lambda F_\lambda^{ab, \nu}. \quad (35)$$

It is easy to see that only the second term on the r.h.s. of (35) contributes to the two-point function in (34). To proceed we need the renormalized, μ -dependent expression for the two-point function (1) in $d = 6$. This is equivalent to obtaining the renormalized expression for the non-integrable singularity $1/x^8$ as $x \rightarrow 0$. The latter is accomplished by the substitution $1/x^8 \rightarrow \mathcal{R}(1/x^8)$ - which corresponds to real-space renormalization [17, 8] - and the use of the general formula [16]

$$\mu \frac{\partial}{\partial \mu} \left[\mathcal{R} \left(\frac{1}{(x^2)^{\frac{1}{2}d+k}} \right) \right] = \frac{\Gamma(\frac{1}{2}d)}{4^k \Gamma(k+1) \Gamma(\frac{1}{2}d+k)} S_d (\partial^2)^k \delta^{(d)}(x). \quad (36)$$

Then, from (1), (34) and (35) we obtain by virtue of (36)

$$\beta_V = \frac{\mathcal{C}_V^{(6)} \pi^3}{960} = \frac{N^3}{192 \pi^3}, \quad (37)$$

where to get the last equality we have used (9) and (31). The result (37) combined with (32) implies that the trace anomaly coefficient β_V is non-renormalized as we go from the weak (free fields) to the strong coupling regime of the (2,0) multiplet in $d = 6$ (up to an overall factor $4N^3$).

Following the same reasoning as above, one expects on general grounds that the properly renormalized three-point function (2) involves certain additional ultralocal structures (e.g. products of delta functions), which are proportional to the two parameters $\mathcal{A}^{(6)}$ and $\mathcal{B}^{(6)}$. This would lead, through (34), to a certain *linear* relationship between $\mathcal{A}^{(6)}$, $\mathcal{B}^{(6)}$ and the coefficient of the conformal anomaly α_V .⁶ A proper analysis of the short-distance singularities in three-point functions will not be attempted here as it is significantly more complicated and has only been achieved in simple cases [8]. Nevertheless, the above arguments imply that the expected relationship between $\mathcal{A}^{(6)}$, $\mathcal{B}^{(6)}$ and α_V , which would follow from such an analysis, should hold both in the strong and the weak coupling regimes of the relevant CFT. If then, as we have shown in (32), the *relative* scale between $\mathcal{A}^{(6)}$ and $\mathcal{B}^{(6)}$ is the same both in the strong and the weak coupling regimes of the (2,0) multiplet in $d = 6$, we conclude that the coefficient α_V remains also the same in both the strong and the weak coupling regimes of the theory (up to the overall factor $4N^3$).

Concluding, we have shown the agreement of the free field theory and $\text{AdS}_7/\text{CFT}_6$ results for the parameters appearing in the two- and three-point functions of the R -symmetry current of the (2,0) multiplet in $d = 6$, up to an overall $4N^3$ factor. The same factor was found in studies of the two- and three-point functions of the energy momentum tensor. Moreover, we argued that such a result implies that the coefficients of the trace anomaly in the presence of external sources for the R -currents are non-renormalized up to the same overall $4N^3$ factor. The trace anomaly is in the same supermultiplet as the R -current anomaly. In the case of $\mathcal{N} = 4$ SYM in $d = 4$, the Adler-Bardeen theorem protects the latter from being renormalized leading to a non-renormalization theorem for the external trace anomaly [15]. In the case of the (2,0) multiplet in $d = 6$, it was found in [5] that some part of the external trace anomaly - the part proportional to the six-dimensional Euler density - *does* renormalize as one goes from the weak to the strong coupling regime. In view of such a result, a better understanding of the renormalization properties of the (2,0) multiplet in $d = 6$ is required. This may be achieved by studies of the R -current anomaly (see [18] for a recent work), using also the approach developed here. Studies of the R -currents in CFTs obtained via $\text{AdS}_4/\text{CFT}_3$ correspondence are also of

⁶Note that only the first term on the r.h.s. of (35) contributes to the three-point function in (34). Similar arguments have been used to connect the external trace anomalies to the parameters in the two- and three-point functions of the energy momentum tensor [8, 5].

great interest, however the absence of a trace anomaly there might lead to less transparent results.

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